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# Influence of Failure Modes of RC Columns on Simplified Seismic Loss Assessment

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**ABSTRACT:** A simplified loss assessment is carried out considering regionally-based design practice for reinforced concrete (RC) buildings. In particular, a 4-story RC moment resisting frame (MRF) is designed according to obsolete seismic Italian design provisions, resulting in a structure characterized by typical features of the building stock in the area struck by the 2009 L'Aquila earthquake. For the same design, two different executive hypotheses are considered, resulting in two non-ductile RC structural configurations dominated by two different failure modes of primary elements, i.e., flexure-shear- and shear- dominated. The loss assessment includes a site-specific hazard analysis, a pushover-based approach for seismic demand assessment, and a simplified damage and cost analyses focused on the RC columns. It is employed to evaluate the risk of earthquake economic losses, and the influence of structural design decisions on these losses.

## 1. INTRODUCTION

Seismic loss assessment represents a main component in the next generation seismic guidelines and codes based on current Performance-Based Earthquake Engineering (PBEE) research. Recent studies have shown the application of the thorough PBEE framework from hazard to loss estimation to different case-study reinforced concrete (RC) structures (e.g., Baradaran Shorake et al., 2013; Ramirez et al., 2012). Some of these studies focus on modern, code-conforming, RC structures in which the capacity design requirements are able to prevent undesirable brittle modes of failure and provide adequate ductility to the building (e.g., Goulet et al., 2007; Ramirez et al., 2012). Similarly, some studies and tools are also available for the application of the complete PBEE methodology to substandard existing RC buildings (e.g., Aslani, 2005; Baradaran Shorake et al., 2013), accounting for non-ductile elements behavior with special focus on columns. In fact, current

PBEE-related methodology for damage and loss estimation employ component-based approach, thus the key issue is the evaluation of damage states and loss functions for single components (e.g., Aslani, 2005; Koduru and Haukaas, 2010).

The occurrence of undesirable modes of failures represents a critical issue in modeling existing RC buildings, and specific modeling options are necessary. For instance, Elwood and Moehle (2003) provided a model able to capture column shear distress and subsequent axial failures; example of loss assessment implementing this modeling approach are available in literature for Californian non-ductile buildings. However, existing buildings do not always conform to a specific standard or construction practice, and typical executive solutions are often a result of local experience (e.g., country- or regionally-based practice).

Available guidelines (and related computer tools) provide a solid state-of-the-art for assessing seismic performance of substandard existing RC buildings; an example are the recent

efforts made by the Applied Technology Council's Project 58 in providing a rich, well-documented methodology, and a related database for implementing PBEE in practice (ATC 58, 2012). On the other hand, there is still an increasing need to progress with country-specific data and tools accounting for local building practice and peculiar characteristics of regional building stocks.

Herein, the focus is on outdated RC Italian design practice; specifically, a 4-story RC MRF is designed according to obsolete seismic Italian design provisions (DM 3/3/1975, 1975), resulting in a structure characterized by typical features of the building stock of the area struck by the M 6.3 2009 L'Aquila earthquake. In fact, in Italy RC structures represent a significant part of residential and commercial building inventories of the last 50 years. Given the same design, two different executive hypotheses are considered, resulting in two structural configurations dominated by two different failure modes, i.e., flexure-shear- and shear- dominated (e.g., De Luca and Verderame, 2015). The considered structures are assumed to be located at a site in the L'Aquila basin ( $42^{\circ}21'14.43''\text{N}$   $13^{\circ}23'31.17''\text{E}$ ). This location was selected as a typical urban Italian site with high levels of seismicity.

A simplified loss estimation methodology, including a site-specific hazard analysis, and a pushover-based simplified approach for seismic demand assessment, is employed to assess the influence of executive building practice on non-ductile RC buildings when it results in different failure modes of primary elements such as first storey columns.

## 2. ARCHETYPE RC BUILDINGS FOR L'AQUILA BUILDING STOCK

The chosen geometry of the representative frame is derived from data collected for the area of Pettino (L'Aquila), and already employed for a case-study analysis and large scale vulnerability studies (Verderame et al., 2011; De Luca et al., 2015). Available data do not include punctual information on seismic details; thus, they were

employed to characterize number of storeys (the typical value for RC buildings in the area is 4 storeys), number and dimensions of bays, and age of construction (most of the RC building stock in the area was built between 1970 and 1980). The selected structure is symmetric in plan, with five bays in longitudinal direction, and three bays in transversal direction. The structural layout is made of parallel plane frames in longitudinal direction only. Interstorey height is equal to 3.2m and bay length is equal to 4.0m, resulting in a floor square surface of  $20 \times 12 \text{ m}^2$ .

A simulated design procedure is performed based on design manuals employed at the time and through the analysis of guidelines employed in Italy at the age of construction, see also Ricci et al. (2011), and Verderame et al. (2014). The assumed age of construction allows identifying the reference codes for gravity (DM 3/10/1978) and for seismic actions (DM 3/3/1975, 1975) used for the design; working stress method is employed for both gravity load and seismic design. A simple lateral load (i.e., linear static) analysis is carried out on the representative longitudinal frame assuming a design spectral acceleration of 0.07g between 0 and 0.8 seconds when using the working stress method (this is equivalent to 0.105g in a limit-states approach), see also Ricci et al., (2011). The above design approach was quite common among professionals at the time.

The simulated design results in beams and columns section dimensions shown in Table 1. Longitudinal reinforcement ratios ( $\rho_l$ ) for columns vary from less than 2% up to less than 3%, with a step reduction from first to fourth storey. Such a  $\rho_l$  value is a typical one for obsolete seismic designed columns in which mostly gravity load dimensioning of elements was carried out and horizontal forces resulted in additional longitudinal reinforcements.

Transversal (i.e., shear) reinforcement according to obsolete codes would result in a stirrup spacing 15 times the lowest longitudinal diameter employed. According to the diameters employed in the longitudinal design the spacing

can vary from 150mm up to 250mm spacing for 8mm diameter stirrups. As in situ inspections in existing buildings often emphasize a lack of seismic detailing, and given the above rule, two different transversal reinforcement hypotheses were made for the RC columns, resulting in two different case study buildings:

- Building A, characterized by 250mm spaced 8mm diameter stirrups as columns' transversal reinforcement, representing a more realistic practical solution based on field inspections carried out after 2009 L'Aquila earthquake and reflecting typical executive building practice at the time;
- Building B, characterized by 150mm spaced  $\phi 8$ mm diameter stirrups as columns' transversal reinforcement, representing an obsolete 'best' design practice.

Table 1: Beams and columns' sections dimensions.

beams	1 <sup>st</sup> and 2 <sup>nd</sup> storey	30x60cm <sup>2</sup>
	3 <sup>rd</sup> and 4 <sup>th</sup> storey	30x50cm <sup>2</sup>
columns (int frame)	1 <sup>st</sup> storey	30x50cm <sup>2</sup>
	2 <sup>nd</sup> storey	30x40cm <sup>2</sup>
	3 <sup>rd</sup> and 4 <sup>th</sup> storey	30x30cm <sup>2</sup>
columns (ext frame)	1 <sup>st</sup> , 2 <sup>nd</sup> , 3 <sup>rd</sup> , and 4 <sup>th</sup> storey	30x30cm <sup>2</sup>

According to the typical material employed at the age of construction, and based on database available in literature, concrete average compressive strength ( $f_{cm}$ ) is assumed equal to 25 MPa, while steel yielding average strength ( $f_{ym}$ ) is assumed equal to 419 MPa, typical of the FeB 38k steel employed at the time.

Elastic linear modeling of the building, assuming uncracked stiffness of elements, results in a fundamental period (T) in the transversal direction equal to 1.1 s and 0.7s in the

longitudinal direction. The structure is evidently a first mode dominated structure with more 75% participant mass ratios in both principal directions.

### 3. SIMPLIFIED LOSS ASSESSMENT

The basis of PBEE approach can be summarized in a single equation (see Eq. 1), Deierlein et al. (2003). In particular, based on the total probability theorem, Eq. 1 allows deconstructing the problem in four steps: (i) hazard analysis, (ii) structural analysis, (iii) damage analysis, and (iv) loss analysis. Each step carries out a specific generalized variable: *Intensity measure (IM)*, *Engineering Demand Parameter (EDP)*, *Damage Measure (DM)*, and *Decision variable (DV)*; in Eq. 1:  $G$  is the complementary cumulative distribution function (CCDF) while  $\lambda$  is the annual rate of exceedance of the considered variables ( $DV$  or  $IM$ ). The key issue of the PBEE methodology is to identify and quantify the DVs of primary interest to the decision makers accounting for all the relevant uncertainties. DVs have been typically defined in terms of different quantities, such as repair costs, downtime, and casualty rates.

The methodology carried out in this paper is based on some simplified assumptions on each step of Eq. (1). The general aim of this numerical application is to provide a preliminary comparison between different construction practices of buildings, given the same design, and consequently between columns' modes of failure in terms of expected annual losses through a straightforward, and practice oriented, approach.

$$\lambda(DV) = \iiint G(DV|DM) dG(DM|EDP) dG(EDP|IM) d\lambda(IM) \quad (1)$$

This simplified loss assessment combines the findings from Aslani (2005) and the approach presented in Koduru and Haukaas (2010), while the ATC 58 (2012) approach, based on nonlinear static analysis, is used to assess the seismic demands. The choice of the intermediate variables are those typical of PBEE loss estimation analysis: the chosen IM is the pseudo spectral acceleration at the fundamental period of

the structure  $S_a(T)$ , the chosen EDP for columns is the interstorey drift ratio (IDR<sub>i</sub>) at each  $i$ -th story, the chosen DMs are based on Aslani's (2005) component fragility functions carried out for columns with light transverse reinforcement, and, finally, the chosen DV is the direct monetary structural loss.

It is worth noting that structural losses generally play a non-significant role in loss estimation of RC buildings given the prominent role played by non-structural components such as sliding windows, ceilings, etc (e.g., Calvi, 2013); on the other hand this simplified approach allows to effectively characterize the influence of different failure modes on the seismic loss of existing RC building.

### 3.1. Hazard analysis

A site-specific hazard analysis accounting for uncertainty in the factors affecting ground motions is carried out. In particular, a Probabilistic Seismic Hazard Analysis (PSHA) is performed by using a Monte Carlo simulation-based approach (e.g., Assatourians and Atkinson, 2013). To this aim, a synthetically generated set of potential earthquakes, with their temporal and geographical distribution, is developed by drawing random samples from the assumed PSHA model components (and related probability distributions), i.e., source-zone geometries and magnitude-recurrence parameters and maximum magnitude. The official Italian seismogenetic zonation, named ZS9 (Meletti et al., 2009), is used in this study; the calculation is limited to events with source-to-site distance up to 150 km (Figure 1). Gutenberg-Richter (GR) parameters implemented for generating each record are adapted from Barani et al., 2010.

The resulting synthetic catalog has a duration of 5,000 years; each record of the synthetic catalog contains the following fields: time (in decimal years), coordinates (latitude and longitude) and magnitude of earthquake, source zone number and corresponding fault-style. In fact, ZS9 assigns a prevalent mechanism of faulting – interpreted as the mechanism with the highest probability of generating future

earthquakes – to all its source zones for use in the Ground Motion Prediction Equations (GMPEs).

Finally, the considered IM for the case-study buildings - i.e.,  $S_a(T)$  - is evaluated for each seismic event contained in the catalog by using the Akkar and Boomer (2010) GMPE assuming soft/stiff soil. A random number drawn from the standard normal distribution is multiplied by the given sigma value (variability of the GMPE model) and added to the median log-IM(from the GMPE) aimed at modeling the aleatory variability in ground motions. The resulting site-specific hazard curves for each event in the catalog as well as the median, 16th and 84th hazard curves are shown in Figure 1.

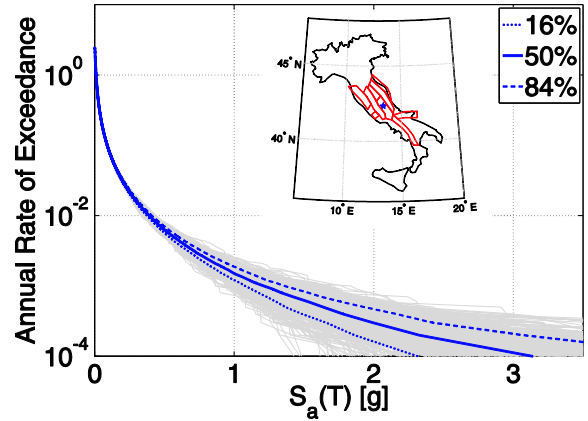


Figure 1: Seismic hazard curve for the case-study site in L'Aquila; considered seismogenetic zones.

### 3.2. Structural analysis

The relationship between EDP and IM is built through nonlinear static analysis, accounting only for the longitudinal direction of Building A and B. The model consist of an exterior frame linked to an interior frame by axially rigid links, such that both type of frames undergo the same lateral displacement. P-Delta effects were disregarded in the structural analysis. Half of the connection dimensions was assumed rigid in beam-column connections.

In literature models accounting accurately for the failure mode of RC elements (e.g., Elwood and Moehle, 2003) are available. Herein, a simplified modeling approach is adopted accounting for both shear-flexure interaction and

loss of vertical load carrying capacity, based on the RC element thresholds carried out on experimental basis by Elwood and Moehle (2003). The axial load failure is included in the moment-rotation (or  $M-\theta$ ) spring without a separate spring accounting for it, as it is stated in the original modeling solution by Elwood and Moehle. This approximation allows to capture in a easy and straightforward way the pre-classified behavior of the RC elements.

The  $M-\theta$  of each element is based on pre-classification of the failure mode through the comparison of the plastic shear ( $V_p$ ) with a degrading shear capacity curve (see Figure 2). The shear capacity model is defined through a linear degrading curve from initial shear capacity ( $V_0$ ), and degraded shear capacity ( $V_d$ ).

- Shear dominated failure (S):  $V_p > V_0$ ; flexural response is modified by a preemptive shear failure in the elastic response phase of the element. Deformation capacity (i.e., drift, chord rotation) is very limited. Subsequent response of the element is characterized by a strictly degrading behavior with significant shear strength reduction and a consequently loss of axial load carrying capacity, see Figure 2a.
- Flexure-shear dominated failure (FS):  $V_0 > V_p > V_d$ ; yielding in the element takes place, flexural response is characterized by an inelastic phase, the inelastic flexural response is modified by a shear failure that occurs at an intermediate

shear strength between  $V_0$  and  $V_d$ , see Figure 2b.

- Flexure dominated failure (F):  $V_d > V_p$ ; yielding in the element takes place, flexural response is not affected by interaction with shear. Thus, the element is characterized by inelastic deformation dominated by flexure, and its typical damage, see Figure 2c.

The pre-classification is made for each element of Building A and B through Sezen and Moehle (2004) shear capacity model. All beams of both Buildings A and B are pre-classified as F elements. In building A, all columns at 1st, 2nd, and 3rd storeys are classified as FS, while 4th storey columns are classified as F. In building B, all columns at 1st, 2nd, and 3rd storeys are classified as S with the exception of those at 1st storey of the external frame classified as FS, finally all columns at 4th storey of both internal and external frames are classified as FS.

Nonlinear modeling is made through a lumped plasticity model characterized by a  $M-\theta$  rotational spring at each end of the element and an elastic frame element modeled with uncracked concrete stiffness. Fixed shear span is assumed equal to half the clear length of each element. Backbones points and drift thresholds are based on literature and code formulations that account implicitly for cyclic degradation (Elwood and Moehle, 2003; Biskinis and Fardis, 2010a; 2010b). Thus, even through a nonlinear static analysis the drift thresholds account for cyclic degradation.

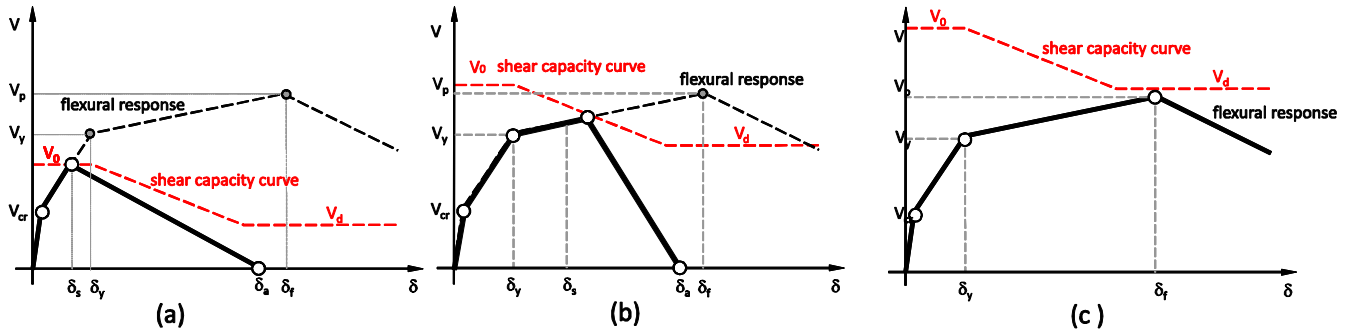


Figure 2: (a) S, (b) FS, and (c) F dominated failures and the assumed backbone for each case

### 3.2.1. Nonlinear static analysis and IDA curves

A conventional (i.e., first-mode) pushover analysis is carried out for each building, considering the only modal distribution. Both building A, and B show a first storey plastic mechanism that involves mostly S elements, and only FS elements in A and B cases, respectively (Figure 3).

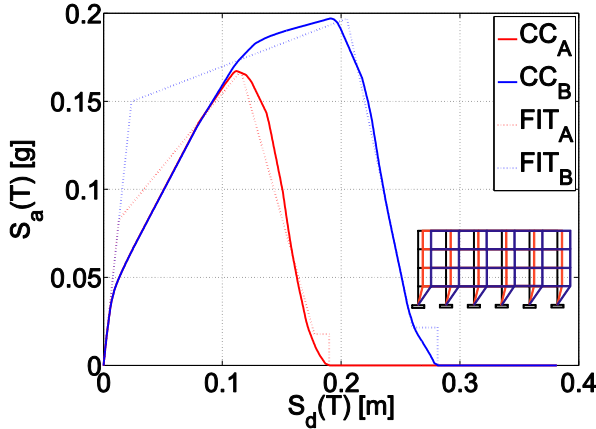


Figure 3: Capacity curves (CC), and piecewise linear fits (FIT) for buildings A, and B.

The pushover curves for the two buildings are then reduced to capacity curves (CC) of the equivalent single degree of freedom (SDOF) systems, and multi-linearized according to the optimized fit by De Luca et al. (2013). SPO2IDA tool (Vamvatsikos and Cornell, 2006) is employed for the evaluation of approximate Incremental Dynamic Analysis (IDA) curves (Figure 4), in analogy with the approach suggested for nonlinear static procedure in the ATC 58. The IDA curves represent the EDP|IM relationship and its variability (see also 16° and 84° percentiles in Figure 4). In fact, there is an univocal correspondence between equivalent SDOF displacement,  $S_d(T)$ , and the  $IDR_i$  of each of the four floors of the two buildings.

### 3.3. Damage Analysis

Damage analysis relates the EDPs to DMs. The DMs include quantitative descriptions of damage to structural elements, nonstructural elements, and contents. The EDPs considered in structural analysis are the input to a set of fragility

functions that model the probability of various level of physical damage, conditioned on structural response. Herein damage analysis is carried out through a simplified approach. First of all, the only elements to which the analysis is applied are columns. All columns in both buildings A and B are studied according to the fragility functions calibrated by Aslani (2005). This approach identifies four damage states (DS) for each column and it can strictly be referred to FS and S columns. In this simplified approach, also given the plastic mechanism observed, the same damage model is applied to columns at 4<sup>th</sup> storey of building B even if they are classified as F.

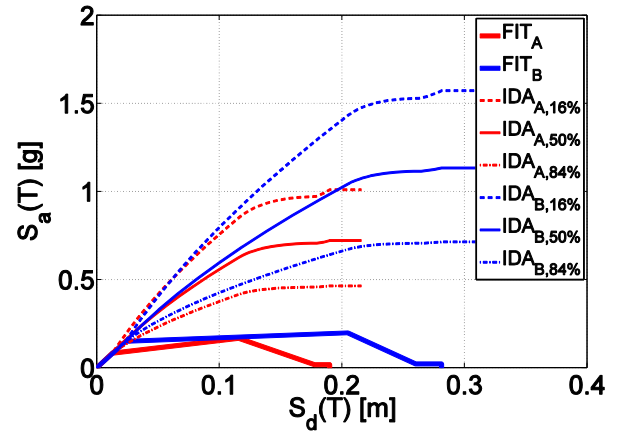


Figure 4: Piecewise linear fits (FIT) and IDA curves for buildings A, and B.

For each column the discriminating parameter  $\alpha = P / (A_g \rho'' f_{cm})$  is evaluated. It is function of  $f_{cm}$ , axial load ( $P$ ), gross area section ( $A_g$ ), and transversal reinforcement ratio ( $\rho''$ ). Given  $\alpha_j$  and  $IDR_i$  for each  $j^{th}$  column of  $i^{th}$  floor of each building the probability of being in a specific damage state (DS) can be evaluated. An example of fragility function relating EDP and damage for 1<sup>st</sup> storey internal column of building B is shown in Figure 5.

### 3.4. Loss analysis

Given the lack of data on repair costs and post-earthquake losses for the specific region considered, some preliminary assumptions are employed in this simplified losses assessment framework.



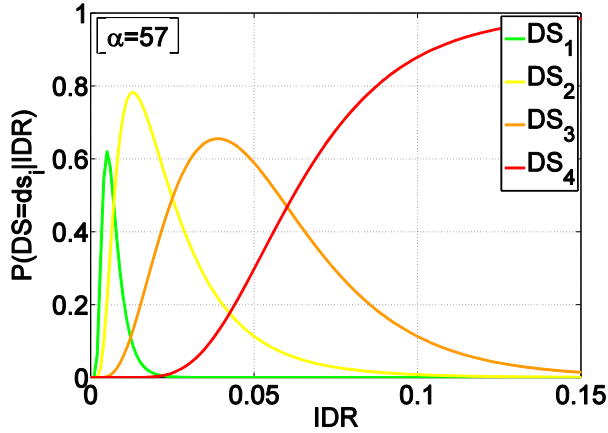


Figure 5: Probability of being in each DS, given the IDR value attained by the column and its  $\alpha$  value for an internal first storey column of building B.

Loss functions, including only direct structural monetary losses, are employed to convert damage to columns in monetary losses. The simplified solution herein is based on the assumption that column damage analysis is sufficient for the approximate evaluation of whole structural losses.

The cost of the whole building (excluding contents) is assumed equal to 1500euro/m<sup>2</sup> based on typical cost for residential buildings in Italy. Structural value is considered equal to 25% of the total cost. This amount then divided for the 96 columns of each building and the equivalent column cost is increased by 20% in order to account for demolition of the damaged component related to the specific content. The attainment of DS<sub>4</sub> (i.e., loss of vertical load carrying capacity) imply the loss of the whole structural value (times the probability of being in DS<sub>4</sub>). In Figure 6, the loss assessment for the two buildings is shown and the two return periods (i.e., 500 and 50 years) typically considered by codes for the limit states of residential buildings are highlighted. The comparison of the two loss curves emphasizes that accounting explicitly for the mode of failure in the structural model can results in different structural losses in the range of return periods typically considered by codes for residential buildings. The simple code-based approach covering few return periods would

have not emphasized any significant difference between the two structural configurations.

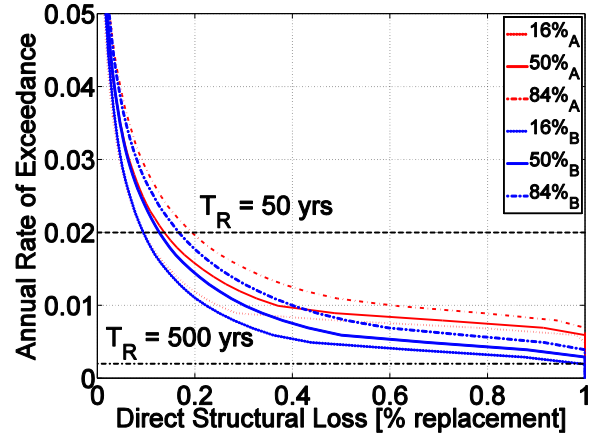


Figure 6: Structural loss assessment for Building A and B.

#### 4. CONCLUSIONS

A practice oriented seismic loss assessment approach is employed to compare structural losses resulting from different failure mode of columns. The results provided represents a first step towards a more general and flexible practice-oriented seismic loss estimation methodology that can be easily implemented through the tools typically employed by professionals in practice and that can, at the same time, increase the level of information for stakeholders.

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